TMUP-HEL-9703
February, 1997

AN ANALYSIS ON A SUPERSYMMETRIC CHIRAL GAUGE THEORY WITH NO FLAT DIRECTION *

N. KITAZAWA †

Department of Physics, Tokyo Metropolitan University,

Hachioji-shi, Tokyo 192-03, Japan

Abstract

A guiding principle to determine the Kähler potential in the low energy effective theory of the supersymmetric chiral gauge theory with no flat direction is proposed. The guiding principle is applied to the $SU(5)$ gauge theory with chiral superfields in the 5^* and 10 representation, and the low energy effective theory is consistently constructed. The spontaneous supersymmetry breaking takes place in the low energy effective theory as expected. The particle mass spectrum in the low energy is explicitly calculated.

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*Talk given at the International Workshop on Perspectives of Strong Coupling Gauge Theories (SCGT96), November 1996, Nagoya, Japan

†e-mail: kitazawa@phys.metro-u.ac.jp

I. INTRODUCTION

It is the long-standing subject to systematically evaluate the non-perturbative effect of the gauge theory in four dimension. In the non-supersymmetric theory we have to use some non-systematic truncation, and only the qualitative feature of the non-perturbative effect is known. On the other hand, a breakthrough occurred in the method of the analysis of the supersymmetric gauge theory. For example, the low energy effective theory of the $N = 2$ supersymmetric $SU(2)$ Yang-Mills theory is exactly determined [1], and the superpotential of the low energy effective theory of the $N = 1$ supersymmetric QCD is also exactly determined [2–4]. The method is also applied to understand the interesting phenomena of the dynamical supersymmetry breaking by the strong gauge interaction, [5–9] and the result is used to construct some concrete models. [10–13]

The supersymmetric chiral gauge theory with no flat direction, especially the $SU(5)$ gauge theory with the chiral superfields in the 5^* and 10 representation, has been extensively considered as the system where the dynamical supersymmetry breaking can be expected. Although the new method is powerful to obtain the low energy effective theory of the $SU(5)$ gauge theory with two pairs of the chiral superfields in the 5^* and 10 representation, [15,16] it is not straightforward to apply to the $SU(5)$ gauge theory with one pair of the chiral superfields in the 5^* and 10 representation, because the effect of the non-trivial Kähler potential is important at low energy. [14] The new method, the principle of the symmetry and holomorphy, is powerful to determine the superpotential, but it does not give any constraint to the Kähler potential.

In this paper we present a guiding principle to determine the effective Kähler potential of the low energy effective theory of the supersymmetric chiral gauge theory with no flat direction. [17] As an example, the low energy effective theory of the the $SU(5)$ gauge theory with the one pair of the chiral superfields in the 5^* and 10 representations is constructed.

II. FUNDAMENTALS OF THE THEORY AND LOW ENERGY EFFECTIVE FIELDS

We first summarize the classical properties of the supersymmetric $SU(5)$ gauge theory with chiral superfields in the 5^* and 10 representations.

The anomaly-free global symmetry of this system is $U(1)_R \times U(1)_A$. The charges of the fields are as follows.

$$\begin{array}{ccc} & U(1)_R & U(1)_A \\ W^{\dot{\alpha}} & -1 & 0 \\ \Phi & 9 & 3 \\ \Omega & -1 & -1 \end{array} \tag{1}$$

Here $W^{\dot{\alpha}}$ is the chiral superfield of the $SU(5)$ gauge field strength, and Φ and Ω denote the chiral superfields of the matter in the 5^* and 10 representations, respectively. The classical scaler potential comes from the D-component of the vector superfields,

$$V_D = \frac{1}{2g^2} D^a D^a \tag{2}$$

with

$$D^a = g^2 \left\{ A_\Phi^\dagger T_{5^*}^a A_\Phi + A_\Omega^\dagger T_{10}^a A_\Omega \right\}, \tag{3}$$

where g denotes the gauge coupling constant, and A_Φ and A_Ω are the scalar components of the chiral superfields Φ and Ω, and $T_{5^*}^a$ and T_{10}^a denote the generators of $SU(5)$ for the 5^* and 10 representations, respectively. It is well known that $A_\Phi = 0$ and $A_\Omega = 0$ is the unique solution of the stationary condition of this potential, and the classical vacuum is supersymmetric.

It is remarkable that no gauge invariant superpotential can be written down. Since all the gauge invariant holomorphic polynomial composed by the chiral superfields Φ and Ω vanish, we can not consider non-trivial superpotential even the non-renormalizable one. This fact means that the gauge coupling g or the scale of the dynamics Λ is a unique parameter in this theory.

Now we consider what effective fields are appropriate in this theory. Since we do not know the symmetry of the exact vacuum unlike in the case of supersymmetric QCD, we must assume it. Here we assume that both $U(1)_R$ and $U(1)_A$ symmetry are spontaneously broken, and there is no massless fermions except for the Nambu-Goldstone fermion due to the spontaneous breaking of supersymmetry. Therefore, 't Hooft anomaly matching condition is not imposed. Furthermore, the confinement of $SU(4)$, a subgroup of $SU(5)$, is assumed rather than the confinement of $SU(5)$ itself. This is the assumption of the complementarity, namely, we will proceed the analysis in Higgs phase rather than the confining phase. These assumptions have to be justified by the result of the analysis.

We introduce only the effective fields which couple with the Lorentz invariant bi-linear operators composed by the three fields Φ, Ω and W in the original theory. In addition, we assume that the effective fields are in the smallest representations of $SU(5)$ in each bi-linear combinations. Namely, we consider the following three effective fields.

$$\begin{array}{lll} X \equiv X^{i=5} & X^i \sim \epsilon^{ijklm}\Omega_{jk}\Omega_{lm} & 10 \times 10 \to 5^* \\ Y \equiv Y_{i=5} & Y_i \sim \Phi^j \Omega_{ji} & 5^* \times 10 \to 5 \\ S & S \sim \mathrm{tr}\left(W^{\dot\alpha} W_{\dot\alpha}\right) & 24 \times 24 \to 1 \end{array} \tag{4}$$

The operator corresponding $5^* \times 5^* \to 10^*$ vanishes, since the superfields commute each other. Since we assume the confinement of $SU(4)$, only the $SU(4)$-singlet parts of each effective fields are introduced as the effective fields.

This procedure is supported by the following arguments. The classical scalar potential eq.(2) can be written like

$$\begin{aligned} V_D &= \frac{g^2}{2}\left[\left(A_\Omega^\dagger T_{10}^a A_\Omega\right)\left(A_\Omega^\dagger T_{10}^a A_\Omega\right) + \cdots\right] \\ &= \frac{g^2}{2}\left[-\lambda(10,10,5^*)\frac{1}{4!}\left|\epsilon A_\Omega A_\Omega\right|^2 - \lambda(10,10,50)\left|(A_\Omega A_\Omega)_{50}\right|^2 + \cdots\right], \end{aligned} \tag{5}$$

where $\lambda(r_1, r_2, r_c) \equiv \{C_2(r_1) + C_2(r_2) - C_2(r_c)\}/2$, and $C_2(r)$ denotes the coefficient of the second Casimir invariant of the representation r of $SU(5)$. [1] The method of the auxiliary

[1]The operator correspond to the channel $10 \times 10 \to 45$ vanishes because of the Bose statistics of

field can be used to introduce the effective fields.

$$
\begin{aligned}
V_D \to V_D &+ \frac{1}{2}\lambda(10,10,5^*)\frac{1}{4!}\mid g\ \epsilon A_\Omega A_\Omega - A_X|^2 + \cdots \\
&= \frac{1}{2}\lambda(10,10,5^*)\frac{1}{4!}A_X^i A_{Xi}^\dagger - \frac{g}{2}\lambda(10,10,5^*)\frac{1}{4!}\left\{(\epsilon A_\Omega A_\Omega)^i A_{Xi}^\dagger + \text{h.c.}\right\} \\
&\quad + \cdots,
\end{aligned} \tag{6}
$$

where A_X^i denotes the scalar component of the effective field X^i. This result shows that if the coefficient λ is positive, the classical squared mass of the effective field becomes positive, and it is worth considering. The effective field in the 5^* representation can be considered, since $\lambda(10,10,5^*) = \frac{12}{5} > 0$, but the effective field in the 50 representation can not be considered, since $\lambda(10,10,50) < 0$, and its classical squared mass is negative. The same arguments are true for the effective fields composed by Φ and Ω. The effective field Y_i is worth considering, since $\lambda(5^*,10,5) = \frac{9}{5} > 0$, but the effective field in the 45 representation can not be considered, since $\lambda(5^*,10,45) < 0$.

From this argument we obtain the classical scalar potential written by the effective fields A_X and A_Y.

$$
V_{\text{classical}} = \lambda_X |A_X|^2 + \lambda_Y |A_Y|^2, \tag{7}
$$

where $\lambda_X \equiv \frac{1}{2}\frac{1}{4!}\lambda(10,10,5^*)$ and $\lambda_Y \equiv \frac{1}{5}\lambda(5^*,10,5)$.

III. SUPERPOTENTIAL AND KÄHLER POTENTIAL

The general form of the superpotential is obtained by the requirement of the symmetry and holomorphy as

$$
W = Sf\left(\frac{\Lambda^{13}}{XYS^3}\right) \tag{8}
$$

with a general holomorphic function f. Note that the power of Λ, 13, which comes from the dimensional analysis, is just the coefficient of the 1-loop β-function of the $SU(5)$ gauge

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the scalar field.

coupling. In the weak coupling limit, $\Lambda \to 0$, this superpotential must coincide with the gauge kinetic term in the perturbatively-calculated Wilsonian action of the original theory. From this condition,

$$W = -\frac{1}{64\pi^2}\ S\ \ln\left(\frac{\Lambda^{13}}{XYS^3}\right) + S\tilde{f}\left(\frac{\Lambda^{13}}{XYS^3}\right), \tag{9}$$

where $\tilde{f}$ is a holomorphic function with $\lim_{z\to 0}\tilde{f}(z) = 0$. Moreover, since we are considering that the massless degrees of freedom are only the Nambu-Goldstone particles, and all of them are described by the effective fields already introduced, the function $\tilde{f}$ should not have the singularities, and it is a constant. The constant is absorbed to the redefinition of Λ. Thus, we obtain

$$W = -\frac{1}{64\pi^2}\ S\ \ln\left(\frac{\Lambda^{13}}{XYS^3}\right). \tag{10}$$

This is the unique superpotential within our postulations.

We propose the following two conditions to constrain the Kähler potential in the effective theory.

1. The Kähler potential coincides with the naive one described by the effective fields in the limit of weak strength of the effective fields,

2. The scalar potential coincides with the classical one in the limit of weak coupling.

The first condition is necessary so that the effective fields are the quantizable fields with canonical kinetic terms. The second condition requires the classical scaler potential which is described by the effective fields like the potential of eq.(7). Here, we demonstrate constructing the non-trivial Kähler potential within the ansatz of factorization:

$$K(X^\dagger X, Y^\dagger Y, S^\dagger S) = K_X(X^\dagger X) + K_Y(Y^\dagger Y) + K_S(S^\dagger S). \tag{11}$$

We can consider the following Kähler potential which satisfies the above two conditions.

$$K_X(X^\dagger X) = \tfrac{1}{\Lambda^2} f(X^\dagger X)_{C_X/\Lambda^4}, \quad K_Y(Y^\dagger Y) = \tfrac{1}{\Lambda^2} f(Y^\dagger Y)_{C_Y/\Lambda^4}, \tag{12}$$

$$K_S(S^\dagger S) = \tfrac{1}{\Lambda^4} S^\dagger S, \tag{13}$$

where C_X and C_Y are real parameters, and a function $f(z)_a$ is defined by

$$f(z)_a \equiv \sum_{n=0}^{\infty}(-1)^n \frac{a^{2n} z^{2n+1}}{(2n+1)^2} = z\ F\left(1, \frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; -a^2 z^2\right). \tag{14}$$

The function F is the generalized hypergeometric function. Note that the Kähler potentials K_X and K_Y become naive ones in the weak field limit of $C_X X^\dagger X/\Lambda^4 \to 0$ and $C_Y Y^\dagger Y/\Lambda^4 \to 0$, respectively (first condition). [2] The scalar potential is obtained as

$$\begin{aligned} V = & \frac{\Lambda^4}{(64\pi^2)^2}\left|\ln\left(\frac{A_X A_Y A_S^3}{\Lambda^{13}}\right)+3\right|^2 + \frac{\Lambda^2}{(64\pi^2)^2}\left(\frac{|A_S|^2}{|A_X|^2}+\frac{|A_S|^2}{|A_Y|^2}\right) \\ & + \frac{C_X^2}{(64\pi^2)^2}\frac{|A_S|^2}{\Lambda^6}|A_X|^2 + \frac{C_Y^2}{(64\pi^2)^2}\frac{|A_S|^2}{\Lambda^6}|A_Y|^2. \end{aligned} \tag{15}$$

The last two terms are the contribution of the non-trivial Kähler potential.

The two parameters C_X and C_Y are determined so that the potential of eq.(15) coincides with the classical one, eq.(7), in $\Lambda \to 0$ limit (second condition). The first two terms of the potential simply vanish in this limit, but the last two terms seem to be singular. We integrate out the effective field S by replacing the field by its vacuum expectation value. The vacuum expectation value of S is proportional to Λ^3, and the coefficient r is independent of Λ, but it depends on C_X and C_Y. Therefore, we can determine these two parameters by the conditions

$$\frac{C_X^2}{(64\pi^2)^2}\, r(C_X, C_Y)^2 = \lambda_X, \quad \frac{C_Y^2}{(64\pi^2)^2}\, r(C_X, C_Y)^2 = \lambda_Y, \tag{16}$$

which are obtained by the second condition. The construction of the effective theory is finished.

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[2] This limit can also be understood as the limit of $\Lambda \to \infty$. If this limit can be regarded as the strong coupling limit, the condition of the coincidence with the naive one may seem to be strange. However, it is not clear whether this limit is really the strong coupling limit, since we do not know the value of the effective coupling below the scale of Λ. In addition, note that "naive" does not mean "tree", namely no quantum correction. The naive Kähler potential is described by the composite effective fields.

IV. VACUUM AND MASS SPECTRUM

The vacuum expectation values of the effective fields are obtained by solving the stationary conditions of the potential of eq.(15). It is analytically shown that all the vacuum expectation values are real and positive, and the numerical calculation gives

$$\langle A_X \rangle \simeq (0.17)^2, \quad \langle A_Y \rangle \simeq (0.11)^2, \quad \langle A_S \rangle \simeq (0.31)^3, \tag{17}$$

in unit of Λ. This solution is consistent with the assumption of breaking $SU(5) \to SU(4)$, since the effective fields X and Y, which are the components of the effective field in the 5^* and 5 representations of $SU(5)$, respectively, obtain the vacuum expectation values. The assumption of the complete breaking of the global $U(1)_R \times U(1)_A$ symmetry is also confirmed. Since the vacuum expectation value of the effective filed S means the gaugino pair condensation, the spontaneous breaking of supersymmetry is expected through Konishi anomaly. [18] In fact, the vacuum energy density is not zero, $V_{\text{vacuum}} \simeq (0.16)^4$ in unit of Λ.

The mass spectrum of the effective fields can be explicitly calculated.

On boson fields, it is convenient to consider the non-linear realization of the global $U(1)_R \times U(1)_A$ symmetry:

$$A_X = \Lambda\ \phi_X e^{i\theta_X/\Lambda}, \quad A_Y = \Lambda\ \phi_Y e^{i\theta_Y/\Lambda}, \quad A_S = \Lambda^2\ \phi_S e^{i\theta_S/\Lambda}, \tag{18}$$

where $\phi_{X,Y,S}$ and $\theta_{X,Y,S}$ are the real scalar fields with dimension one. The eigenvalues of the mass matrix for the real scalar fields $\theta_{X,Y,S}$ are analytically obtained. Two of three eigenvalues are zero which are corresponding to the Nambu-Goldstone bosons of $U(1)_R$ and $U(1)_A$ breaking, and remaining eigenvalue is $m_\theta^2 = 22\Lambda^2/(64\pi^2)^2 \simeq (0.0074\Lambda)^2$. The smallness of this value can be understood by considering that it is corresponding to the mass of the pseudo-Nambu-Goldstone boson due to the anomalous global $U(1)$ symmetry breaking. The eigenvalues of the mass matrix for the fields $\phi_{X,Y,S}$ are numerically obtained as

$$m_\phi^2 \simeq (0.45)^2, \quad (0.73)^2, \quad (1.5)^2, \tag{19}$$

in unit of Λ.

The mass matrix of the fermion components, $\Lambda\psi_X$, $\Lambda\psi_Y$ and $\Lambda^2\psi_S$, of the effective chiral superfields, X, Y and S, respectively, is analytically obtained, where all ψ's have dimension 3/2. One can analytically check that the mass matrix has one zero eigenvalue, which is corresponding to the mass of the Nambu-Goldstone fermion of the supersymmetry breaking, by using the stationary conditions of the scalar potential. The other two eigenvalues are numerically given by

$$m_\psi \simeq 0.33, \quad 0.091, \tag{20}$$

in unit of Λ.

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